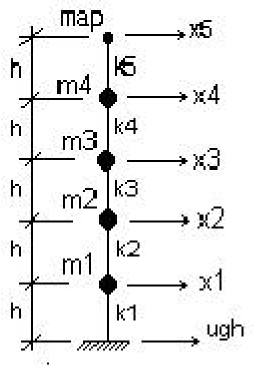
Analisis modal espectral de un modelo de 4 niveles con apendice sometido al Terremoto de Mexico 1985 SCT- 00 grados.



Datos geometricos :

Reference:C:\ING\Unidades\unidades.mcd

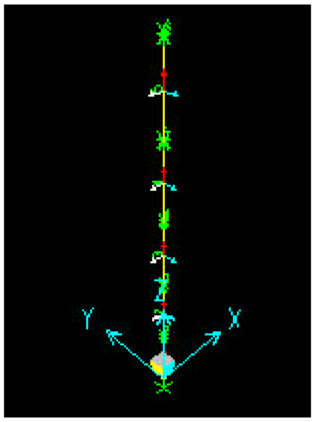
$np := 5$

$mp := 100 \cdot \frac{kip}{g}$ $\quad mp = 4.534 \times 10^{4} \frac{kgf}{g}$

$map := 0.01 \cdot mp$ $\quad map = 453.438 \frac{kgf}{g}$

$kp := 22.599 \cdot \frac{kip}{in}$ $\quad kp = 4.034 \times 10^{3} \frac{kgf}{cm}$

$kap := 0.0012 \cdot kp$ $\quad kap = 4.841 \frac{kgf}{cm}$



Modelo de elementos finitos

Altura de cada piso :

$h := 12 \cdot ft$

$h = 3.658\, m$

Armado de la matriz de masas :

$$M := \begin{pmatrix} mp & 0 & 0 & 0 & 0 \\ 0 & mp & 0 & 0 & 0 \\ 0 & 0 & mp & 0 & 0 \\ 0 & 0 & 0 & mp & 0 \\ 0 & 0 & 0 & 0 & map \end{pmatrix}$$

$$M = \begin{pmatrix} 4.534 \times 10^{4} & 0 & 0 & 0 & 0 \\ 0 & 4.534 \times 10^{4} & 0 & 0 & 0 \\ 0 & 0 & 4.534 \times 10^{4} & 0 & 0 \\ 0 & 0 & 0 & 4.534 \times 10^{4} & 0 \\ 0 & 0 & 0 & 0 & 453.438 \end{pmatrix} \frac{kgf}{g}$$

Armado de la matriz de rigidez :

$$K := \begin{pmatrix} 2 \cdot kp & -kp & 0 & 0 & 0 \\ -kp & 2 \cdot kp & -kp & 0 & 0 \\ 0 & -kp & 2 \cdot kp & -kp & 0 \\ 0 & 0 & -kp & kp + kap & -kap \\ 0 & 0 & 0 & -kap & kap \end{pmatrix}$$

$$K = \begin{pmatrix} 8.069\times10^{5} & -4.034\times10^{5} & 0 & 0 & 0 \\ -4.034\times10^{5} & 8.069\times10^{5} & -4.034\times10^{5} & 0 & 0 \\ 0 & -4.034\times10^{5} & 8.069\times10^{5} & -4.034\times10^{5} & 0 \\ 0 & 0 & -4.034\times10^{5} & 4.039\times10^{5} & -484.121 \\ 0 & 0 & 0 & -484.121 & 484.121 \end{pmatrix} \frac{kgf}{m}$$

Calculo de las frecuencias naturales de vibracion :

$$\omega := \overrightarrow{\sqrt{genvals(K, M)}}$$

$$\omega = \begin{pmatrix} 17.555 \\ 14.312 \\ 9.343 \\ 3.347 \\ 3.135 \end{pmatrix} \frac{rad}{seg}$$

Armado del vector de influencia :

$$\iota := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Reordenamiento de menor a mayor :

$$\omega := sort(\omega)$$

$$\omega = \begin{pmatrix} 3.134908 \\ 3.347481 \\ 9.34301 \\ 14.311777 \\ 17.555285 \end{pmatrix} \frac{rad}{seg}$$

Calculo de los periodos de cada modo :

$$i := 1 .. np$$

$$T_i := \frac{2\cdot\pi}{\omega_i}$$

$$T = \begin{pmatrix} 2.004265 \\ 1.876989 \\ 0.672501 \\ 0.439022 \\ 0.357908 \end{pmatrix} seg$$

Valores muy altos para edificios de este tipo, elegidos a proposito para acentuar las contribuciones de los modos superiores a la respuesta.

Calculo de los modos naturales de vibracion :

$$\phi s := genvecs(K, M)$$

$$\phi s = \begin{pmatrix} 0.428 & 0.656 & 0.576 & 0.025 & -0.021 \\ -0.656 & -0.228 & 0.575 & 0.046 & -0.039 \\ 0.577 & -0.577 & -5.233\times10^{-4} & 0.062 & -0.053 \\ -0.228 & 0.429 & -0.576 & 0.07 & -0.061 \\ 8.023\times10^{-3} & -0.023 & 0.078 & -0.994 & -0.996 \end{pmatrix}$$

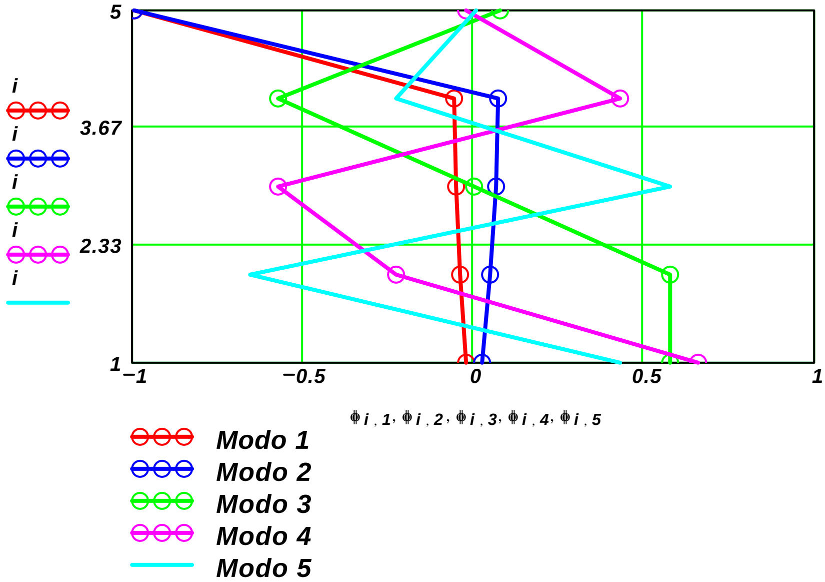
Reordenamiento de los modos moviendo las columnas :

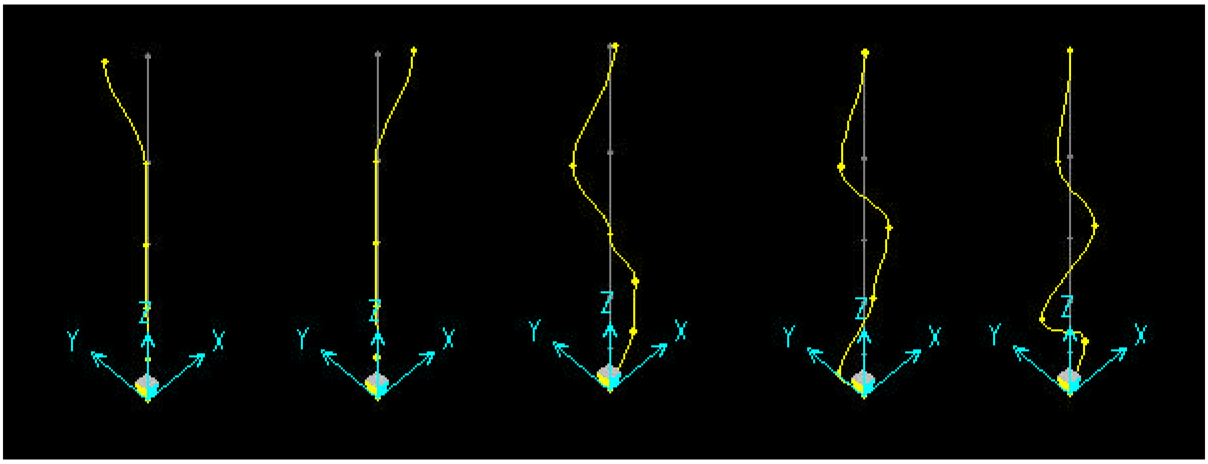
$i := 1 .. np$

$\phi^{\langle i \rangle} := \phi s^{\langle (np+1)-i \rangle}$

$$\phi = \begin{pmatrix} -0.021 & 0.025 & 0.576 & 0.656 & 0.428 \\ -0.039 & 0.046 & 0.575 & -0.228 & -0.656 \\ -0.053 & 0.062 & -5.233\times 10^{-4} & -0.577 & 0.577 \\ -0.061 & 0.07 & -0.576 & 0.429 & -0.228 \\ -0.996 & -0.994 & 0.078 & -0.023 & 8.023\times 10^{-3} \end{pmatrix}$$

Graficacion de los modos de vibracion :





Modo 1 **Modo 2** **Modo 3** **Modo 4** **Modo 5**

Cálculo del factor de excitación símica modal :

$$i := 1 .. np$$

$$\phi^{T} \cdot M \cdot \iota = \begin{pmatrix} -8.345 \times 10^{3} \\ 8.766 \times 10^{3} \\ 2.609 \times 10^{4} \\ 1.268 \times 10^{4} \\ 5.5 \times 10^{3} \end{pmatrix} \frac{kgf}{g}$$

Calculo de la masa generalizada modal :

$$Mg := \phi^{T} \cdot M \cdot \phi$$

$$Mg = \begin{pmatrix} 835.696 & 1.262 \times 10^{-12} & -9.007 \times 10^{-13} & -1.137 \times 10^{-12} & 1.018 \times 10^{-12} \\ 1.298 \times 10^{-12} & 970.239 & -1.026 \times 10^{-12} & -6.048 \times 10^{-13} & 1.144 \times 10^{-13} \\ -6.960 \times 10^{-13} & -8.994 \times 10^{-13} & 4.507 \times 10^{4} & 6.035 \times 10^{-12} & -2.683 \times 10^{-12} \\ -1.071 \times 10^{-12} & -8.689 \times 10^{-13} & 3.598 \times 10^{-12} & 4.532 \times 10^{4} & -6.013 \times 10^{-12} \\ 8.728 \times 10^{-13} & 2.699 \times 10^{-13} & -2.977 \times 10^{-12} & -6.138 \times 10^{-12} & 4.534 \times 10^{4} \end{pmatrix} \frac{kgf}{g}$$

Cálculo de la masa total :

$$MT := mp + mp + mp + mp + map \qquad MT = 1.818 \times 10^{5} \frac{kgf}{g}$$

Cálculo de la masa efectiva modal :

$$\gamma_i := \frac{\left(\left|\phi^{\langle i \rangle T} \cdot M \cdot \iota\right|\right)^2}{Mg_{i,i} \cdot MT} \qquad \gamma = \begin{pmatrix} 0.458 \\ 0.436 \\ 0.083 \\ 0.02 \\ 3.67 \times 10^{-3} \end{pmatrix}$$

Verificacion :

$$\sum_{i=1}^{np} \gamma_i = 1$$

Calculo del factor de participacion modal :

$$L^{\langle i \rangle} := \frac{\phi^{\langle i \rangle T} \cdot M \cdot \iota}{Mg_{i,i}} \qquad L = \begin{pmatrix} -9.985 & 9.035 & 0.579 & 0.28 & 0.121 \end{pmatrix}$$

Distribucion espacial de fuerzas de cada modo :

$$s^{\langle i \rangle} := M \cdot \phi^{\langle i \rangle} \cdot L^{\langle i \rangle}$$

$$s = \begin{pmatrix} 956.985 & 1.037 \times 10^{3} & 1.54 \times 10^{3} & 848.856 & 240.339 \\ 1.806 \times 10^{3} & 1.941 \times 10^{3} & 1.54 \times 10^{3} & -295.002 & -368.236 \\ 2.452 \times 10^{3} & 2.596 \times 10^{3} & -1.401 & -746.334 & 323.855 \\ 2.822 \times 10^{3} & 2.917 \times 10^{3} & -1.541 \times 10^{3} & 554.374 & -127.96 \\ 459.718 & -415.327 & 2.1 & -0.299 & 0.045 \end{pmatrix} kgf$$

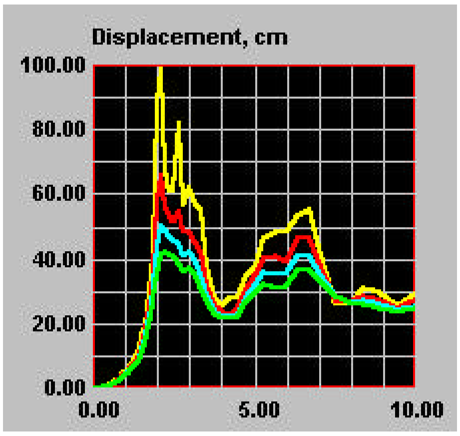
Verificacion :

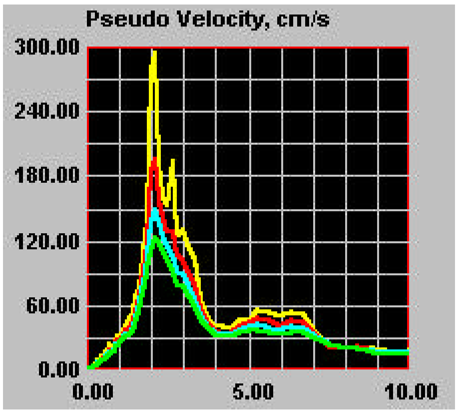
$$M \cdot \iota = \begin{pmatrix} 4.534 \times 10^4 \\ 4.534 \times 10^4 \\ 4.534 \times 10^4 \\ 4.534 \times 10^4 \\ 453.438 \end{pmatrix} \frac{kgf}{g}$$

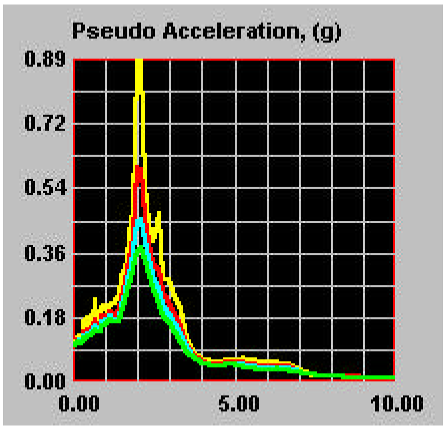
$$\sum_{i=1}^{np} \left(L^{\langle i \rangle} \cdot \phi^{\langle i \rangle^T} \cdot M \right) = \begin{pmatrix} 4.534 \times 10^4 & 4.534 \times 10^4 & 4.534 \times 10^4 & 4.534 \times 10^4 & 453.438 \end{pmatrix} \frac{kgf}{g}$$

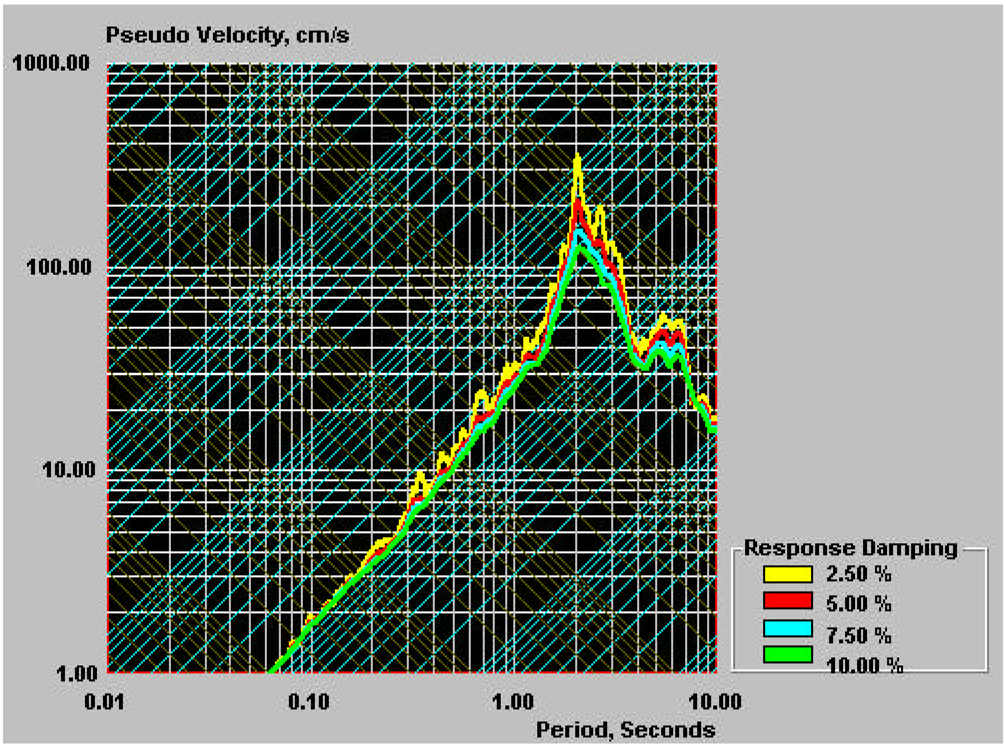
Pseudodesplazamientos del espectro de respuesta del sismo de Mexico (para factor de amortiguamiento 5%) :

$$Sd := \begin{pmatrix} 61 \\ 34.99 \\ 1.61 \\ 0.665 \\ 0.32 \end{pmatrix} \cdot cm$$









Maximos desplazamientos de cada piso para cada modo :

$$u^{\langle i\rangle} := \phi^{\langle i\rangle} \cdot L^{\langle i\rangle} \cdot Sd_i$$

$$u = \begin{pmatrix} 12.625 & 7.848 & 0.536 & 0.122 & 0.017 \\ 23.828 & 14.689 & 0.536 & -0.042 & -0.025 \\ 32.348 & 19.643 & -0 & -0.107 & 0.022 \\ 37.223 & 22.074 & -0.537 & 0.08 & -0.009 \\ 606.492 & -314.295 & 0.073 & -0.004 & 0 \end{pmatrix} cm$$

Calculo de las fuerzas estaticas equivalentes para cada modo :

$$fs^{\langle i\rangle} := s^{\langle i\rangle} \cdot (\omega_i)^2 \cdot Sd_i$$

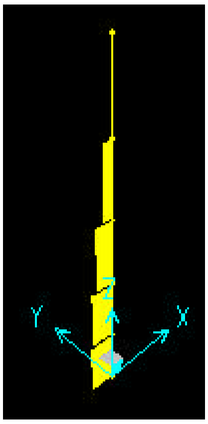
$$fs = \begin{pmatrix} 5.737\times 10^3 & 4.066\times 10^3 & 2.165\times 10^3 & 1.156\times 10^3 & 237.022 \\ 1.083\times 10^4 & 7.611\times 10^3 & 2.164\times 10^3 & -401.822 & -363.154 \\ 1.47\times 10^4 & 1.018\times 10^4 & -1.968 & -1.017\times 10^3 & 319.386 \\ 1.691\times 10^4 & 1.144\times 10^4 & -2.166\times 10^3 & 755.113 & -126.194 \\ 2.756\times 10^3 & -1.628\times 10^3 & 2.952 & -0.407 & 0.044 \end{pmatrix} kgf$$

Comprobacion de calcular los desplazamientos cargando con el vector de fuerzas equivalentes :

$$ust := K^{-1} \cdot fs$$

$$ust = \begin{pmatrix} 12.625 & 7.848 & 0.536 & 0.122 & 0.017 \\ 23.828 & 14.689 & 0.536 & -0.042 & -0.025 \\ 32.348 & 19.643 & -4.877 \times 10^{-4} & -0.107 & 0.022 \\ 37.223 & 22.074 & -0.537 & 0.08 & -8.856 \times 10^{-3} \\ 606.492 & -314.295 & 0.073 & -4.295 \times 10^{-3} & 3.114 \times 10^{-4} \end{pmatrix} cm$$

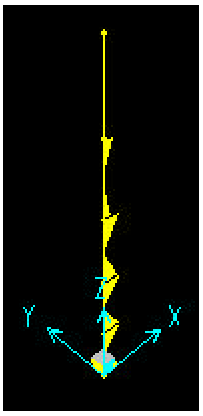
Calculo de los esfuerzos de corte en la base para cada modo de vibracion :



$$Vb_{1,i} := \sum_{l=1}^{np} fs_{l,i}$$

$$Vb = (50.934 \quad 31.663 \quad 2.164 \quad 0.493 \quad 0.067)\, Ton\_fuerza$$

Calculo de los momentos flexores en la base para cada modo de vibracion :



$$H := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \cdot h \qquad Mb_{1,i} := \sum_{l=1}^{np} fs_{l,i} \cdot H_l$$

$$Mb = (559.349 \quad 319.771 \quad -7.908 \quad 1.175 \quad -0.131)\, Ton\_fuerza \cdot m$$

CALCULO DE LA COMBINACION MODAL DE LAS RESPUESTAS :

ABSSUM

$$VbT := \sum_{l=1}^{np} \left|Vb_{1,l}\right| \qquad MbT := \sum_{l=1}^{np} \left|Mb_{1,l}\right| \qquad u5T := \sum_{l=1}^{np} \left|u_{5,l}\right| \qquad u4T := \sum_{l=1}^{np} \left|u_{4,l}\right|$$

$$VbT = 8.532 \times 10^{4}\, kgf \qquad MbT = 8.883 \times 10^{5}\, kgf \cdot m \qquad u5T = 9.209\, m \qquad u4T = 59.923\, cm$$

SRSS

$$VbT := \sqrt{\sum_{l=1}^{np} (Vb_{1,l})^2} \quad MbT := \sqrt{\sum_{l=1}^{np} (Mb_{1,l})^2} \quad u5T := \sqrt{\sum_{l=1}^{np} (u_{5,l})^2} \quad u4T := \sqrt{\sum_{l=1}^{np} (u_{4,l})^2}$$

$$VbT = 6.002 \times 10^4\,kgf \quad MbT = 6.444 \times 10^5\,kgf \cdot m \quad u5T = 6.831\,m \quad u4T = 43.28\,cm$$

CQC

Factores de amortiguamiento para cada modo : $\xi := \begin{pmatrix} 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \end{pmatrix}$

Relaciones de frecuencias naturales :

$i := 1..np$

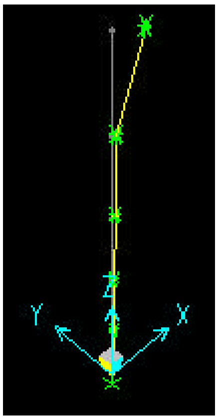
$j := 1..np$

$$\beta_{i,j} := \frac{\omega_i}{\omega_j}$$

$$\beta = \begin{pmatrix} 1 & 0.936 & 0.336 & 0.219 & 0.179 \\ 1.068 & 1 & 0.358 & 0.234 & 0.191 \\ 2.98 & 2.791 & 1 & 0.653 & 0.532 \\ 4.565 & 4.275 & 1.532 & 1 & 0.815 \\ 5.6 & 5.244 & 1.879 & 1.227 & 1 \end{pmatrix} \qquad \omega = \begin{pmatrix} 3.135 \\ 3.347 \\ 9.343 \\ 14.312 \\ 17.555 \end{pmatrix} \frac{rad}{seg}$$

Coeficientes de correlacion :

$$\rho_{i,j} := \frac{8 \cdot \sqrt{\xi_i \cdot \xi_j} \cdot (\xi_i + \beta_{i,j} \cdot \xi_j) \cdot (\beta_{i,j})^{\frac{3}{2}}}{\left[1 - (\beta_{i,j})^2\right]^2 + 4 \cdot \xi_i \cdot \xi_j \cdot \beta_{i,j} \cdot \left[1 + (\beta_{i,j})^2\right] + 4 \cdot \left[(\xi_i)^2 + (\xi_j)^2\right] \cdot (\beta_{i,j})^2}$$



$$\rho = \begin{pmatrix} 1.000 & 0.699 & 6.543 \times 10^{-3} & 2.748 \times 10^{-3} & 1.893 \times 10^{-3} \\ 0.699 & 1.000 & 7.602 \times 10^{-3} & 3.112 \times 10^{-3} & 2.129 \times 10^{-3} \\ 6.543 \times 10^{-3} & 7.602 \times 10^{-3} & 1.000 & 0.050 & 0.023 \\ 2.748 \times 10^{-3} & 3.112 \times 10^{-3} & 0.050 & 1.000 & 0.192 \\ 1.893 \times 10^{-3} & 2.129 \times 10^{-3} & 0.023 & 0.192 & 1.000 \end{pmatrix}$$

$$VbT := \sqrt{\sum_{i=1}^{np} \sum_{j=1}^{np} \rho_{i,j} \cdot Vb^{\langle i \rangle} \cdot Vb^{\langle j \rangle}} \qquad MbT := \sqrt{\sum_{i=1}^{np} \sum_{j=1}^{np} \rho_{i,j} \cdot Mb^{\langle i \rangle} \cdot Mb^{\langle j \rangle}}$$

$$VbT = \left(7.654 \times 10^4\right) kgf \qquad MbT = \left(8.155 \times 10^5\right) kgf \cdot m$$

$$u5T := \sqrt{\sum_{i=1}^{np} \sum_{j=1}^{np} \rho_{i,j} \cdot u_{5,i} \cdot u_{5,j}} \qquad u4T := \sqrt{\sum_{i=1}^{np} \sum_{j=1}^{np} \rho_{i,j} \cdot u_{4,i} \cdot u_{4,j}}$$

$$u5T = 4.475\,m \qquad u4T = 54.962\,cm$$