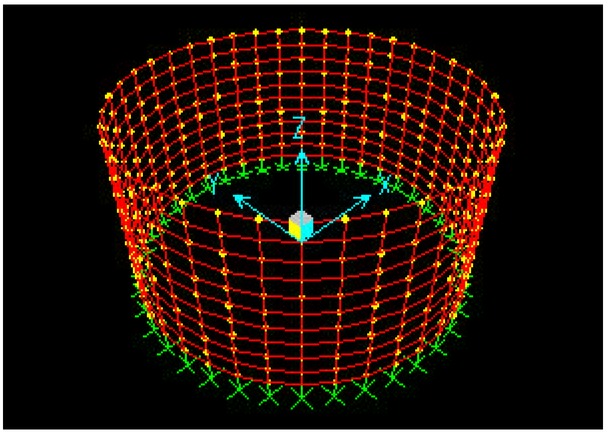
Modelo de tanque cilindrico empotrado en su base con carga de peso propio, presion hidrostatica y temperatura.



Reference:C:\ING\Unidades\unidades.mcd

Datos geometricos y mecanicos :

$H := 7m$

$h := 30cm$

$R := 7m$

$\gamma b := 24 \cdot \frac{KN}{m^3}$

$Ti := 40celsius$

$Te := 20celsius$

$Eb := 30000 \cdot \frac{MN}{m^2}$

$\gamma := 10 \cdot \frac{KN}{m^3}$

$\nu := 0.20$

$\lambda := 1 \cdot 10^{-5} \cdot \frac{1}{celsius}$

1.- PESO PROPIO

Para el caso de peso propio se considera : $\nu pp := 0$

1.1. TENSIONES TEORIA MEMBRANAL

Propiedades de la cascara :

$R\psi = \infty$ Radio de curvatura del meridiano

$R\phi := R$ Distancia medida sobre la normal desde la curva hasta el eje

$\psi := 90grados$

Ecuaciones de equilibrio :

$$Pnpp := 0 \cdot \frac{KN}{m^2}$$

$$Peppa(a) := \gamma b \cdot h \cdot 2 \cdot \pi \cdot R \cdot a$$

Pe es la resultante de todas las fuerzas verticales que son sostenidas por ***Tx*** para un ***e*** determinado (corto con cun plano normal al eje por la coordenada ***e*** dada)

$$T\phi pp := R\phi \cdot Pnpp$$

$$Txppa(a) := \frac{-Peppa(a)}{2 \cdot \pi \cdot R \cdot sin(\psi)} \qquad Txpp(x) := \frac{-Peppa(H - x)}{2 \cdot \pi \cdot R \cdot sin(\psi)}$$

1.2. DEFORMACIONES TEORIA MEMBRANAL

$$\varepsilon xppa(a) := \frac{1}{Eb \cdot h} \cdot (Txppa(a) - \nu pp \cdot T\phi pp) \qquad \varepsilon xpp(x) := \varepsilon xppa(H - x)$$

$$\varepsilon \phi ppa(a) := \frac{1}{Eb \cdot h}(-\nu pp \cdot Txppa(a) + T\phi pp) \qquad \varepsilon \phi pp(x) := \varepsilon \phi ppa(H - x)$$

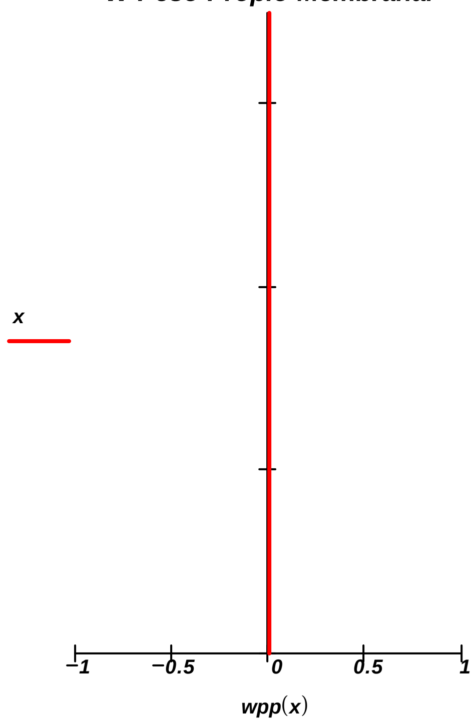
$\varepsilon \phi = \frac{w}{R}$ entonces $wppa(a) := R \cdot \varepsilon \phi ppa(a)$

$x(a) := H - a$ $\qquad wpp(x) := R \cdot \varepsilon \phi pp(H - x)$

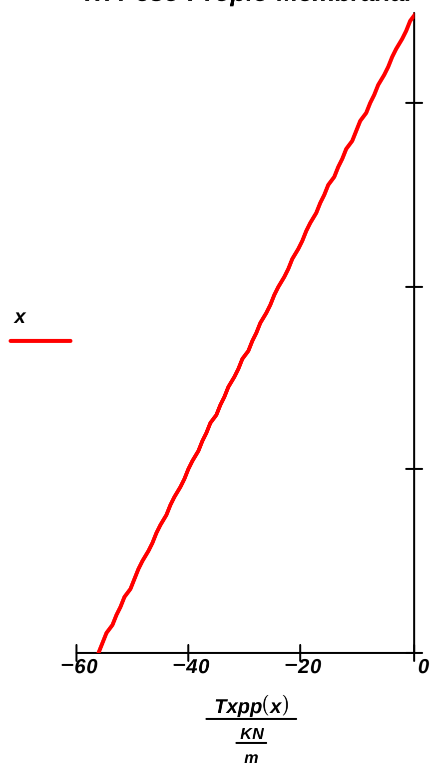
$\theta = \frac{dw}{dx}$ entonces $\theta ppa(a) := \frac{d}{da} wppa(a)$

$x := 0m, 0.1m .. H$

W Peso Propio Membranal



Tx Peso Propio Membranal



$$Txpp(0m) = -56.376 \frac{KN}{m}$$

Para todo **x** se cumple que ***w(x)=0*** , ***Mx(x) = 0*** , ***Mϕ(x) = 0*** , ***Tϕ(x) = 0***

2.- PRESION HIDROSTATICA

2.1. TENSIONES TEORIA MEMBRANAL

Ecuaciones de equilibrio :

$$Pnpha(a) := \gamma \cdot a$$

$$Peph(a) := 0KN$$

Pe es la resultante de todas las fuerzas verticales que son sostenidas por ***Tx*** para un ***e*** determinado (corto con cun plano normal al eje por la coordenada ***e*** dada)

$$T\phi pha(a) := R\phi \cdot Pnpha(a) \qquad T\phi phx(x) := T\phi pha(H - x)$$

$$Txpha(a) := \frac{-Peph(a)}{2\pi R \cdot \sin(\psi)} \qquad Txphx(x) := \frac{-Peph(H - x)}{2\pi R \cdot \sin(\psi)}$$

2.2. DEFORMACIONES TEORIA MEMBRANAL

$$\varepsilon xpha(a) := \frac{1}{Eb \cdot h} \cdot (Txpha(a) - \nu \cdot T\phi pha(a)) \qquad \varepsilon xphx(x) := \varepsilon xpha(H - x)$$

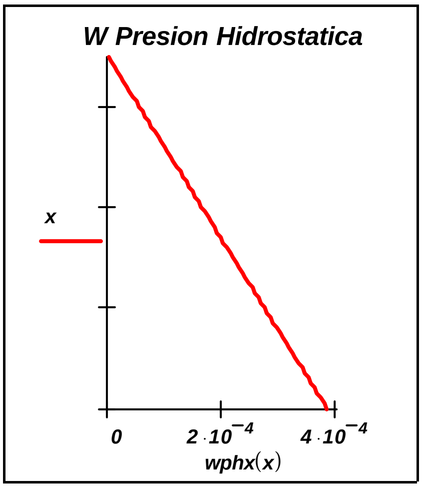
$$\varepsilon\phi pha(a) := \frac{1}{Eb \cdot h}(-\nu \cdot Txpha(a) + T\phi pha(a)) \qquad \varepsilon\phi phx(x) := \varepsilon\phi pha(H - x)$$

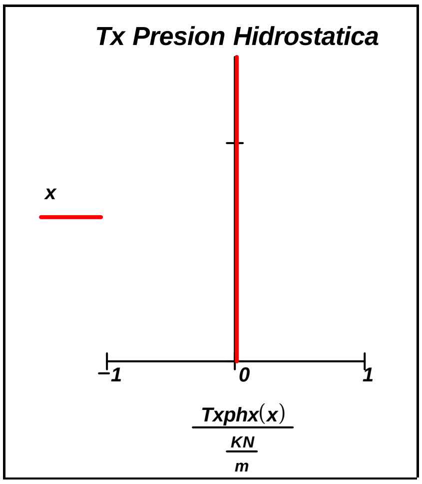
$\varepsilon\phi = \frac{w}{R}$ entonces $\qquad wpha(a) := R \cdot \varepsilon\phi pha(a)$

$x(a) := H - a \qquad wphx(x) := R \cdot \varepsilon\phi pha(H - x)$

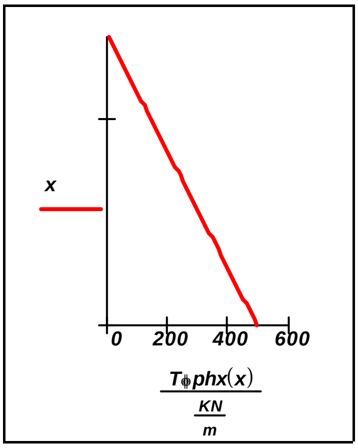
$\theta = \frac{dw}{dx}$ entonces $\qquad \theta phx(x) := \frac{d}{dx} wphx(x)$

$$x := 0m, 0.1m .. H$$





Tϕ Presion Hidrostatica



$$wphx(0m) = 0.381\,mm \qquad Txphx(0m) = 0\frac{KN}{m} \qquad T\phi phx(0m) = 490\frac{KN}{m}$$

Para ***x=0*** , el desplazamiento debido a la presion hidrostatica es distinto de cero, con lo cual no cumple con las condiciones de borde. Es necesario introducir esfuerzos de flexion.

2.3. ANALISIS DE LA PERTURBACION EN EL BORDE INFERIOR

Se debe determinar el valor de Mo y Qo, tal que se cumplan las condiciones de borde.
Se resuelve por flexibilidades

$$D := \frac{Eb \cdot h^3}{12 \cdot (1 - \nu^2)} \qquad D = 70.312\, MN \cdot m \qquad \beta := \sqrt[4]{\frac{Eb \cdot h}{4 \cdot D \cdot R^2}} \qquad \beta = 0.899 \frac{1}{m}$$

$$a_{11} := \frac{-1}{2 \cdot \beta^3 \cdot D} \cdot \frac{KN}{m} \qquad a_{11} = -9.789 \times 10^{-6}\, m$$

$$a_{21} := \frac{1}{2 \cdot \beta^2 \cdot D} \cdot \frac{KN}{m} \qquad a_{21} = 8.8 \times 10^{-6}$$

$$a_{12} := \frac{-1}{2 \cdot \beta^2 \cdot D} \cdot \frac{KN \cdot m}{m} \qquad a_{12} = -8.8 \times 10^{-6}\, m$$

$$a_{22} := \frac{1}{\beta \cdot D} \cdot \frac{KN \cdot m}{m} \qquad a_{22} = 1.582 \times 10^{-5}$$

$$wo := wphx(0 \cdot m) \qquad wo = 0.381\, mm$$

$$\theta o := \theta phx(0 \cdot m) \qquad \theta o = -5.444 \times 10^{-5}$$

$$TI := \begin{pmatrix} wo \\ \theta o \end{pmatrix} \qquad Matriz := \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad soln := lsolve(Matriz, -TI)$$

$$soln = \begin{pmatrix} 71.681 \\ -36.428 \end{pmatrix} \quad \text{en KN}$$

$$Qoph := soln_1 \cdot \frac{KN}{m} \qquad Qoph = 71.681 \frac{KN}{m}$$

$$Moph := soln_2 \cdot \frac{KN}{m} \qquad Moph = -36.428 \frac{KN \cdot m}{m}$$

Superposicion de efectos : desplazamiento de teoria membranal + desplazamiento Mo, Qo

$$wphx'(x) := \frac{\gamma \cdot R^2 \cdot (H - x)}{Eb \cdot h} + \frac{e^{-\beta \cdot x}}{2 \cdot \beta^3 \cdot D} \cdot [\beta \cdot Moph \cdot (\sin(\beta \cdot x) - \cos(\beta \cdot x)) - Qoph \cdot \cos(\beta \cdot x)]$$

$$\theta phx'(x) := \frac{d}{dx} wphx'(x)$$

El primer termino corresponde al desplazamiento debido a la presion hidrostatica (solucion particular de la ecuacion diferencial) y el segundo termino es el desplazamiento debido a ***Mo, Qo*** (solucion homogenea)

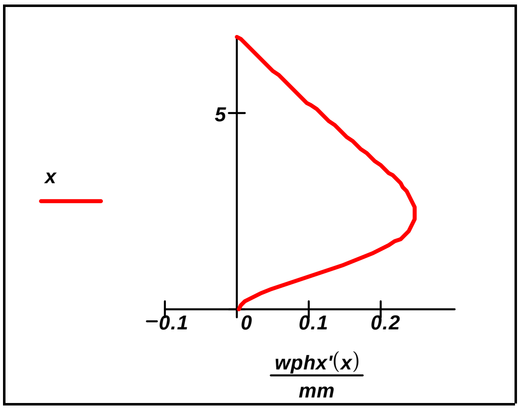
$$T\phi phx'(x) := \frac{Eb \cdot h \cdot wphx'(x)}{R} \qquad Txphx'(x) := 0 \cdot \frac{KN}{m}$$

$$Mxphx'(x) := \frac{e^{-\beta \cdot x}}{\beta} \cdot [\beta \cdot Moph \cdot (sin(\beta \cdot x) + cos(\beta \cdot x)) + Qoph \cdot sin(\beta \cdot x)]$$

$$M\phi phx'(x) := \nu \cdot Mxphx'(x)$$

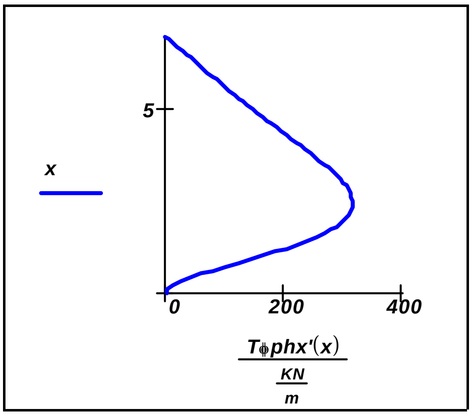
$$Qxphx'(x) := e^{-\beta \cdot x} \cdot [(-2 \cdot \beta \cdot Moph \cdot sin(\beta \cdot x)) + Qoph \cdot (cos(\beta \cdot x) - sin(\beta \cdot x))]$$

Desplazamientos wphx'(x)



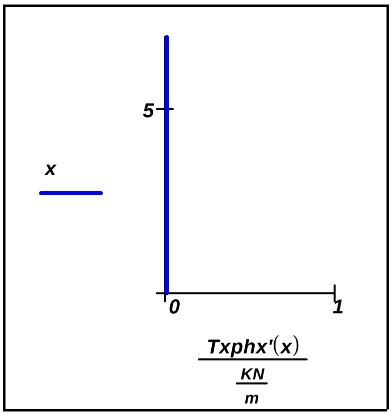
$$wphx'(2.5 \cdot m) = 0.244\,mm$$

Esfuerzos Tϕphx'(x)

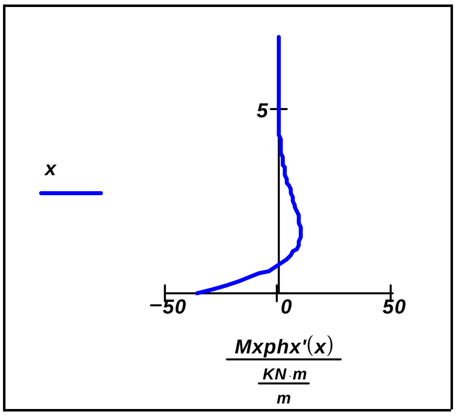


$$T\phi phx'(2.4 \cdot m) = 313.683 \frac{KN}{m}$$

Esfuerzos Txphx'(x)



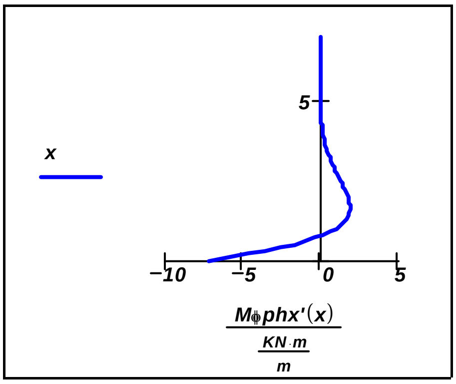
Momentos flexores Mxphx'(x)

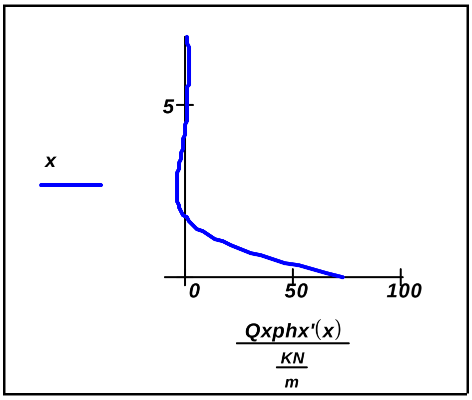


$$Mxphx'(1.7 \cdot m) = 9.05\,KN \cdot \frac{m}{m}$$

$$Mxphx'(0 \cdot m) = -36.428\,KN \cdot \frac{m}{m}$$

Momentos flexores Mϕphx'(x)





$$M\phi phx'(1.7 \cdot m) = 1.81 \frac{KN \cdot m}{m}$$

$$Qxphx'(2.7 \cdot m) = -5.139 \frac{KN}{m}$$

$$M\phi phx'(0 \cdot m) = -7.286 \frac{KN \cdot m}{m}$$

$$Qxphx'(0 \cdot m) = 71.681 \frac{KN}{m}$$

Si H fuese menor, entonces al introducir una perturbacion en ***x = 0,*** en ***x = H*** el momento y el corte no son nulos, pero ese borde es libre, por lo cual se produce una incompatibilidad; es necesario entonces plantear una perturbacion en el borde superior e inferior simultaneamente (que se acoplen). La perturbacion viaja mas lejos cuanto mayor es el radio y el espesor.

3.- TEMPERATURA

3.1. TENSIONES TEORIA MEMBRANAL

Como existe un salto termico, no es posible descomponer en estado membranal y ***Mo*** y ***Qo*** que permitan cumplir con las condiciones de borde. se considera la solucion particular para zonas alejadas de los bordes

$$To := \frac{Te + Ti}{2} \qquad To = 30\,celsius$$

$$\Delta T := Te - Ti \qquad \Delta T = -20\,celsius$$

$$pT := \frac{Eb \cdot h}{R} \cdot \lambda \cdot To \qquad wT := \frac{pT}{4 \cdot \beta^4 \cdot D}$$

Se considera la expresion simplificada con ***Tx = 0*** (libremente dilatable)

$$T\phi T := Eb \cdot h \cdot \left(\frac{wT}{R} - \lambda \cdot To \right) \qquad T\phi T = 0 \frac{KN}{m} \quad \text{sin esfuerzos}$$

$$MxT := -D \cdot \lambda \cdot (1 + \nu) \cdot \frac{\Delta T}{h} \qquad MxT = 56.25 \frac{KN \cdot m}{m}$$

$$M\phi T := -D \cdot \lambda \cdot (1 + \nu) \cdot \frac{\Delta T}{h} \qquad M\phi T = 56.25 \frac{KN \cdot m}{m}$$

$$woT := wT$$

$$woT = 2.1\,mm$$

$$\theta oT := 0$$

3.2. PERTURBACION DE BORDE

3.2.1 BORDE INFERIOR

Vale la solucion homogenea. La matriz de flexibilidad es la misma, cambia ***wo*** y θo. Luego se obtienen las expresiones de momento, tensiones y corte, como suma de la solucion particular y la homogenea.

$$Matriz = \begin{pmatrix} -9.789 \times 10^{-6} & -8.8 \times 10^{-6} \\ 8.8 \times 10^{-6} & 1.582 \times 10^{-5} \end{pmatrix} \qquad TlT := \begin{pmatrix} \frac{woT}{m} \\ \theta oT \end{pmatrix}$$

$$soln := lsolve(Matriz, -TlT) \qquad soln = \begin{pmatrix} 429.069 \\ -238.649 \end{pmatrix}$$

$$QoT := soln_1 \cdot \frac{KN}{m} \qquad QoT = 429.069 \frac{KN}{m}$$

$$MoT := soln_2 \cdot \frac{KN \cdot m}{m} \qquad MoT = -238.649 \frac{KN \cdot m}{m}$$

Superposicion de efectos : desplazamiento teoria membranal + desplazamiento Mo, Qo

$$wT'(x) := woT + \frac{e^{-\beta \cdot x}}{2 \cdot \beta^3 \cdot D} \cdot [\beta \cdot MoT \cdot (sin(\beta \cdot x) - cos(\beta \cdot x)) - QoT \cdot cos(\beta \cdot x)]$$

$$\theta T'(x) := \frac{d}{dx} wT'(x)$$

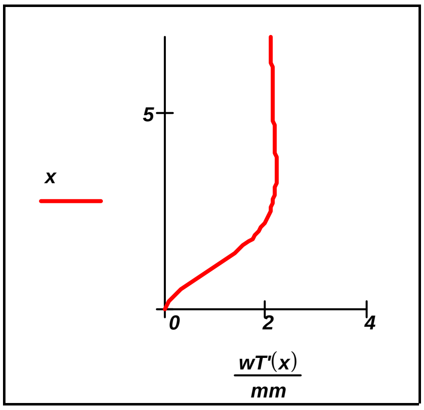
$$T\phi T'(x) := Eb \cdot h \cdot \left(\frac{wT'(x)}{R} - \lambda \cdot To \right) \qquad TxT'(x) := 0 \cdot \frac{KN}{m}$$

$$MxT'(x) := \frac{e^{-\beta \cdot x}}{\beta} \cdot [\beta \cdot MoT \cdot (sin(\beta \cdot x) + cos(\beta \cdot x)) + QoT \cdot sin(\beta \cdot x)] - D \cdot \lambda \cdot (1 + \nu) \cdot \frac{\Delta T}{h}$$

$$M\phi T'(x) := -D \cdot \left[\nu \cdot \frac{d^2}{dx^2} wT'(x) + \lambda \cdot (1 + \nu) \cdot \frac{\Delta T}{h} \right]$$

$$QxT'(x) := e^{-\beta \cdot x} \cdot [(-2 \cdot \beta \cdot MoT \cdot sin(\beta \cdot x)) + QoT \cdot (cos(\beta \cdot x) - sin(\beta \cdot x))]$$

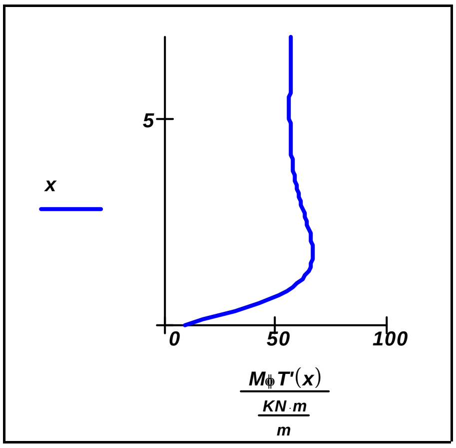
Desplazamientos wT'(x)



$$wT'(3.5\cdot m) = 2.191\,mm$$

$$wT'(7\cdot m) = 2.096\,mm$$

Momentos flexores MϕT'(x)

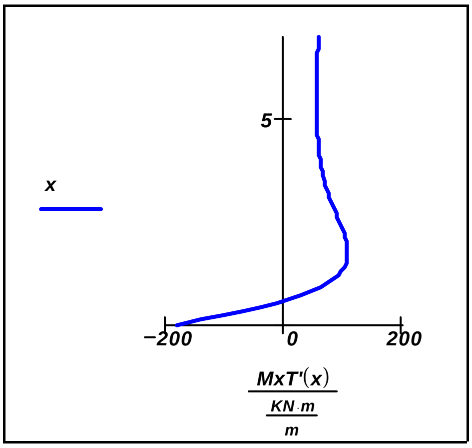


$$M\phi T'(1.7\cdot m) = 66.154\,\frac{KN\cdot m}{m}$$

$$M\phi T'(7\cdot m) = 56.163\,\frac{KN\cdot m}{m}$$

$$M\phi T'(0\cdot m) = 8.52\,\frac{KN\cdot m}{m}$$

Momentos flexores MxT'(x)

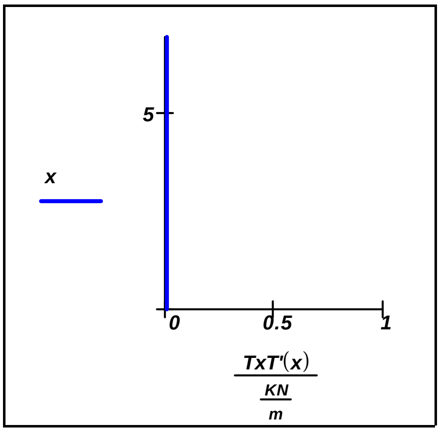


$$MxT'(1.8\cdot m) = 105.753\,\frac{KN\cdot m}{m}$$

$$MxT'(7\cdot m) = 55.813\,\frac{KN\cdot m}{m}$$

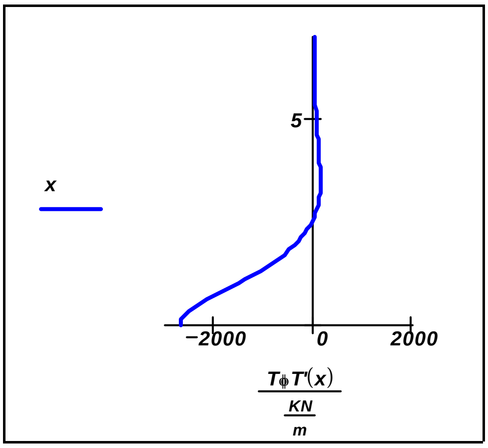
$$MxT'(0\cdot m) = -182.399\,\frac{KN\cdot m}{m}$$

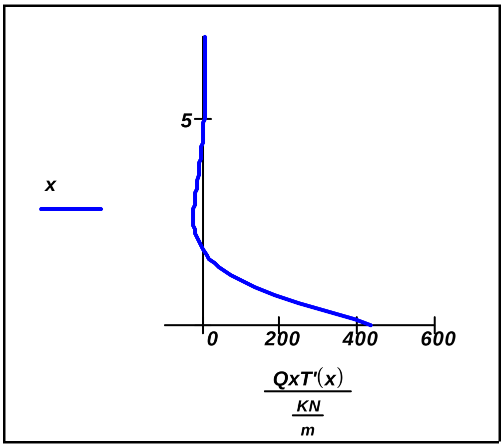
Esfuerzos TxT'(x)



Esfuerzos TϕT'(x)

Esfuerzos de corte QxT'(x)





$$T\phi T'(3.5\cdot m) = 116.675\,\frac{KN}{m}$$

$$QxT'(2.6\cdot m) = -28.746\,\frac{KN}{m}$$

$$T\phi T'(0\cdot m) = -2700\,\frac{KN}{m}$$

$$QxT'(0\cdot m) = 429.069\,\frac{KN}{m}$$

3.2.2 BORDE SUPERIOR

El borde superior es libre, por lo cual ***Q(x=H) =0*** y ***M(x=H) = 0***

$$QxT'(H) = 0.794\,\frac{KN}{m}$$ El corte es practicamente nulo

$$M1 := -MxT'(H) \qquad M1 = -55.813\,\frac{KN\cdot m}{m}$$

Se debe aplicar un momento en el extremo libre igual a ***M1*** tal que la suma, anula el momento en dicho extremo

$$wBSa(a) := \frac{e^{-\beta\cdot a}}{2\cdot\beta^3\cdot D}\cdot\beta\cdot M1\cdot(\sin(\beta\cdot a) - \cos(\beta\cdot a))$$

$$T\phi BSa(a) := Eb\cdot h\cdot\frac{wBSa(a)}{R} \qquad TxBSa(a) := 0\cdot\frac{KN}{m}$$

$$MxBSa(a) := e^{-\beta\cdot a}\cdot M1\cdot(\sin(\beta\cdot a) + \cos(\beta\cdot a))$$

$$M\phi BSa(a) := \nu\cdot e^{-\beta\cdot a}\cdot M1\cdot(\sin(\beta\cdot a) + \cos(\beta\cdot a))$$

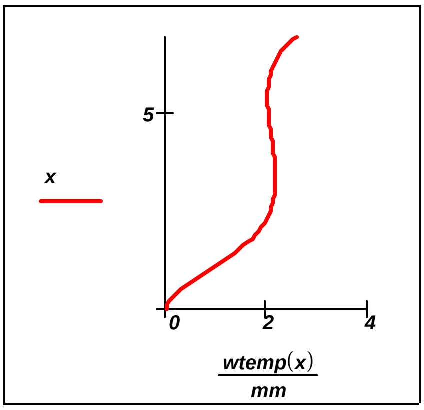
$$QxBSa(a) := e^{-\beta\cdot a}\cdot 2\cdot\beta\cdot M1\cdot\sin(\beta\cdot a)$$

$$wtemp(x) := wT'(x) + wBSa(H - x) \qquad Mxtemp(x) := MxT'(x) + MxBSa(H - x)$$

$$T\phi temp(x) := T\phi T'(x) + T\phi BSa(H - x) \qquad M\phi temp(x) := M\phi T'(x) + M\phi BSa(H - x)$$

$$Txtemp(x) := TxT'(x) + TxBSa(H - x) \qquad Qxtemp(x) := QxT'(x) + QxBSa(a)$$

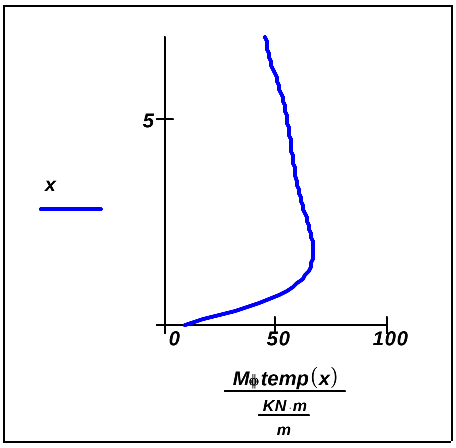
Desplazamientos wtemp(x)



$wtemp(3.4\cdot m) = 2.173\,mm$

$wtemp(7\cdot m) = 2.587\,mm$

Momentos flexores Mϕtemp(x)

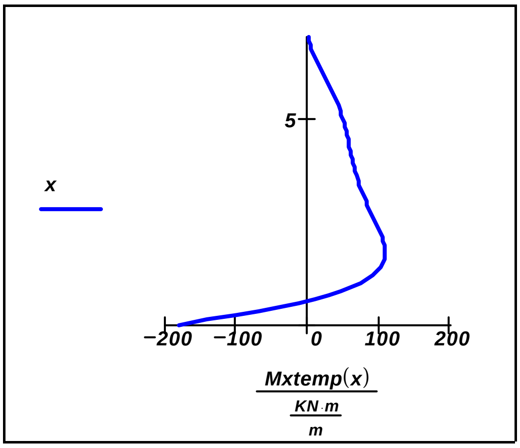


$M\phi temp(7\cdot m) = 45\,\frac{KN\cdot m}{m}$

$M\phi temp(1.8\cdot m) = 66.259\,\frac{KN\cdot m}{m}$

$M\phi temp(0\cdot m) = 8.499\,\frac{KN\cdot m}{m}$

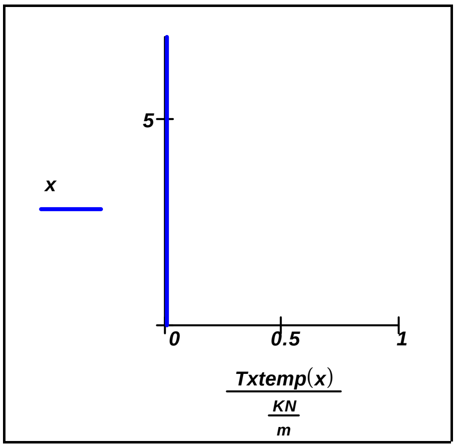
Momentos flexores Mxtemp(x)



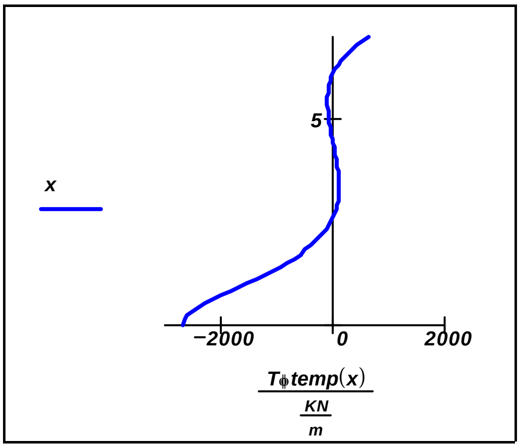
$Mxtemp(1.8\cdot m) = 106.293\,\frac{KN\cdot m}{m}$

$Mxtemp(0\cdot m) = -182.503\,\frac{KN\cdot m}{m}$

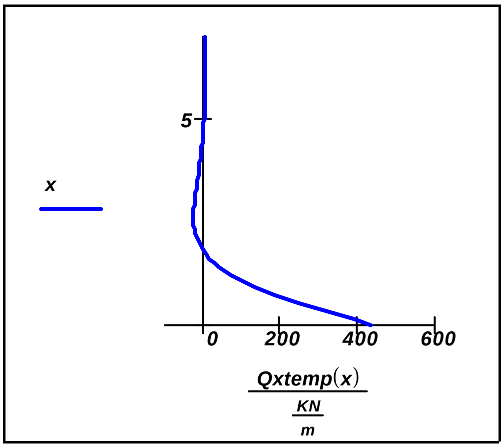
Esfuerzos Txtemp(x)



Esfuerzos Tϕtemp(x)



Esfuerzos de corte Qxtemp(x)



$$T\phi temp(7\cdot m) = 626.408\,\frac{KN}{m}$$

$$T\phi temp(0\cdot m) = -2698.843\,\frac{KN}{m}$$

$$Qxtemp(2.7\cdot m) = -28.618\,\frac{KN}{m}$$

$$Qxtemp(0\cdot m) = 429.069\,\frac{KN}{m}$$

DEFORMACIONES Y SOLICITACIONES TOTALES

$$wf(x) := wpp(x) + wphx'(x) + wtemp(x)$$

$$T\phi f(x) := T\phi phx'(x) + T\phi temp(x)$$

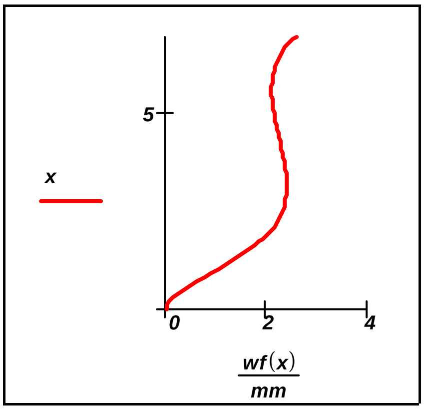
$$Txf(x) := Txpp(x) + Txphx'(x) + Txtemp(x)$$

$$Mxf(x) := Mxphx'(x) + Mxtemp(x)$$

$$M\phi f(x) := M\phi phx'(x) + M\phi temp(x)$$

$$Qxf(x) := Qxphx'(x) + Qxtemp(x)$$

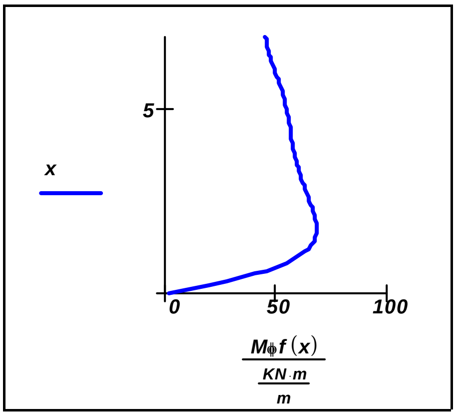
Desplazamientos wf(x)



$$wf(3.2\cdot m) = 2.395\,mm$$

$$wf(7\cdot m) = 2.586\,mm$$

Momentos flexores Mϕf(x)

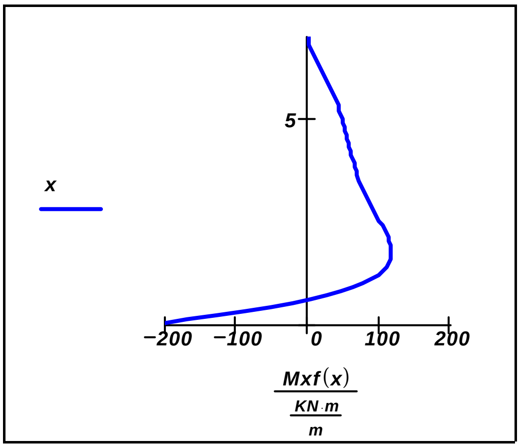


$$M\phi f(7\cdot m) = 44.987\,\frac{KN\cdot m}{m}$$

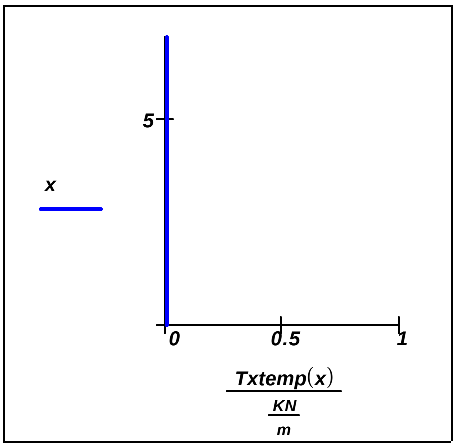
$$M\phi f(1.8\cdot m) = 68.042\,\frac{KN\cdot m}{m}$$

$$M\phi f(0\cdot m) = 1.214\,\frac{KN\cdot m}{m}$$

Momentos flexores Mxf(x)



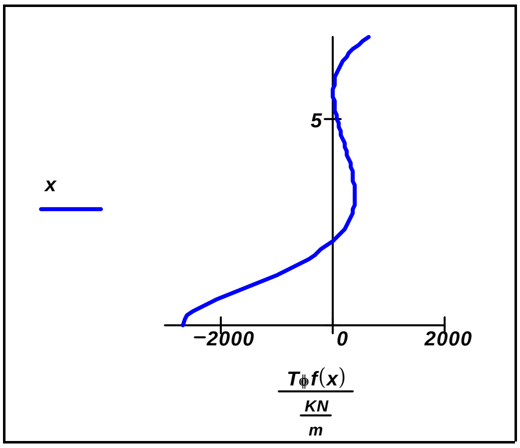
Esfuerzos Txf(x)



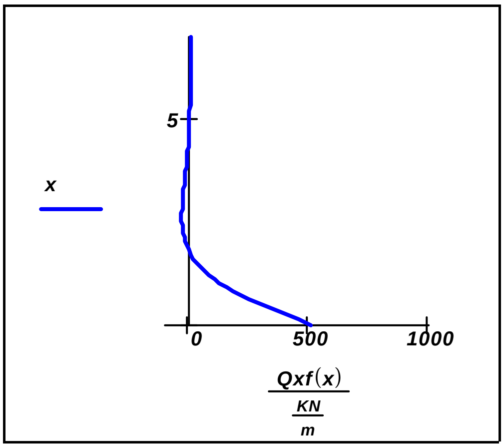
$$Mxf(1.8 \cdot m) = 115.212 \frac{KN \cdot m}{m}$$

$$Mxf(0 \cdot m) = -218.93 \frac{KN \cdot m}{m}$$

Esfuerzos Tϕf(x)



Esfuerzos de corte Qxf(x)



$$T\phi f(7 \cdot m) = 625.494 \frac{KN}{m}$$

$$T\phi f(0 \cdot m) = -2698.843 \frac{KN}{m}$$

$$Qxf(2.7 \cdot m) = -33.757 \frac{KN}{m}$$

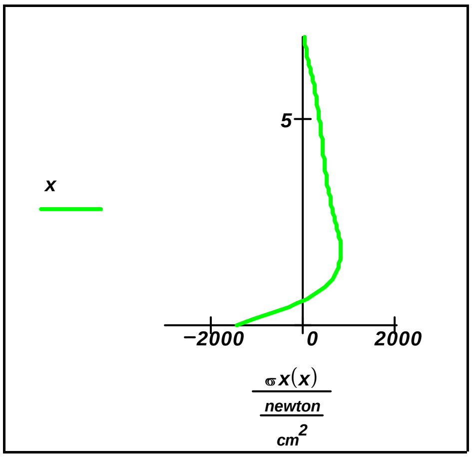
$$Qxf(0 \cdot m) = 500.749 \frac{KN}{m}$$

TENSIONES

Se determinan las tensiones en la cara superior considerando al material como homogeneo

$$\sigma x(x) := 6 \cdot \frac{Mxf(x)}{h^2} \qquad \sigma\phi 1(x) := \frac{T\phi f(x)}{h} + 6 \cdot \frac{M\phi f(x)}{h^2} \qquad \sigma\phi 2(x) := \frac{T\phi f(x)}{h} - 6 \cdot \frac{M\phi f(x)}{h^2}$$

Tensiones σx(x)

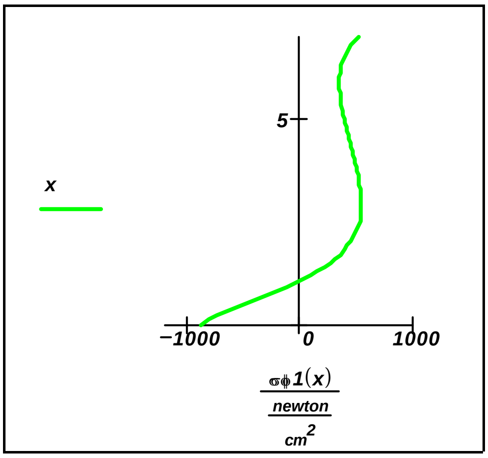


$$\sigma x(1.7 \cdot m) = 768.455 \frac{newton}{cm^2}$$

$$\sigma x(0 \cdot m) = -1459.536 \frac{newton}{cm^2}$$

$$\sigma x(7 \cdot m) = -0.444 \frac{newton}{cm^2}$$

Tensiones σφ1(x)

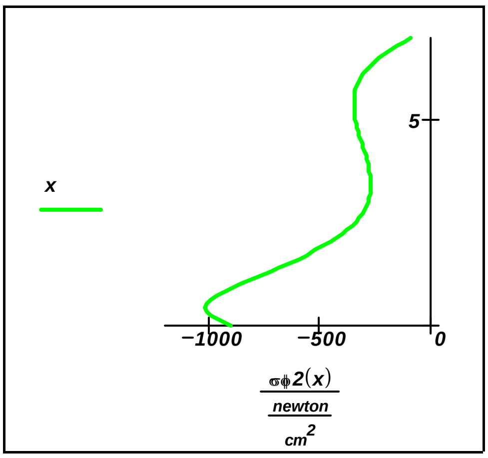


$$\sigma\phi 1(7 \cdot m) = 508.409 \frac{newton}{cm^2}$$

$$\sigma\phi 1(2.9 \cdot m) = 537.351 \frac{newton}{cm^2}$$

$$\sigma\phi 1(0 \cdot m) = -891.522 \frac{newton}{cm^2}$$

Tensiones σφ2(x)



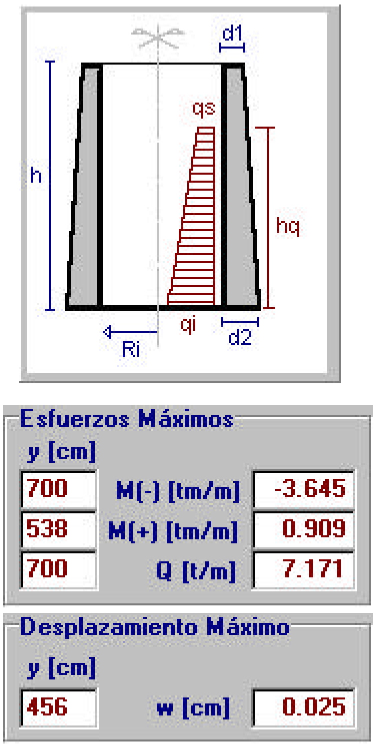
$$\sigma\phi 1(7 \cdot m) = 508.409 \frac{newton}{cm^2}$$

$$\sigma\phi 1(5.5 \cdot m) = 347.884 \frac{newton}{cm^2}$$

$$\sigma\phi 1(0.5 \cdot m) = -477.697 \frac{newton}{cm^2}$$

$$\sigma\phi 1(0 \cdot m) = -891.522 \frac{newton}{cm^2}$$

Resolucion del tanque cilindrico para el caso hidrostatico :



Prof [m]	M[tm/m]	Q[t/m]	w [cm]	T_ϕ[t/m]
0.00	0.000	-0.000	0.000	0.365
0.28	-0.002	-0.008	-0.001	-1.721
0.56	-0.007	-0.019	-0.003	-3.809
0.84	-0.014	-0.034	-0.005	-5.906
1.12	-0.022	-0.053	-0.006	-8.021
1.40	-0.030	-0.077	-0.008	-10.165
1.68	-0.036	-0.108	-0.010	-12.349
1.96	-0.038	-0.147	-0.011	-14.581
2.24	-0.035	-0.197	-0.013	-16.863
2.52	-0.022	-0.259	-0.015	-19.190
2.80	0.004	-0.335	-0.017	-21.539
3.08	0.047	-0.424	-0.019	-23.873
3.36	0.110	-0.524	-0.020	-26.128
3.64	0.196	-0.629	-0.022	-28.211
3.92	0.305	-0.728	-0.023	-29.995
4.20	0.436	-0.806	-0.024	-31.321
4.48	0.581	-0.839	-0.025	-31.996
4.76	0.724	-0.796	-0.025	-31.812
5.04	0.843	-0.636	-0.024	-30.562
5.32	0.900	-0.312	-0.022	-28.077
5.60	0.848	0.225	-0.019	-24.280
5.88	0.621	1.027	-0.015	-19.256
6.16	0.145	2.130	-0.010	-13.348
6.44	-0.669	3.551	-0.006	-7.265
6.72	-1.907	5.262	-0.002	-2.209
7.00	-3.645	7.171	-0.000	-0.000